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Theory of the hatchet planimeter

SIMON FARTHING

Department of Mathematics, Simon Fraser University, Burnaby B C, V5A 1S6, Canada

Present address: 975 Tuam Road, RR 1 Sidney B C, V8L 3R9, Canada

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Abstract. The remarkable area measurement property of the hatchet planimeter is proved directly and easily by a series expansion. The previously known correction terms are readily generated and extended by a new analysis which now easily shows any blade misalignment to be of minor effect. This allows rationalizing the design for adjustment to give the best precision for the size of the figure. A new operating scheme, illustrated by an example, attempts to ensure accuracy at the minimum of user sophistication and calculation, to realize the hatchet planimeter's enormous advantage in cost, mechanical simplicity and ruggedness over the Amsler planimeter. This is the background to an attempt to popularize this planimeter for rural engineering (Farthing *et al* 1985).

Keywords. Area; planimeter; hatchet; Prytz.

1. Introduction

The standard analog instrument for measuring irregularly shaped areas, such as those enclosed by contours on a map, curves on a plan, or underneath a graphed function, is the polar planimeter invented by Jakob Amsler in 1856. In it a wheel vernier—graduated to 0·001 of a turn—is moved obliquely to its axis, so that it partly slips over the paper and partly rotates. This necessitates a delicate needle point bearing in an exacting alignment, and a fine worm gearing to count complete revolutions; explaining the £165 starting price in 1982.

In stark mechanical and price contrast is the hatchet planimeter of no moving parts at all. It was in 1886 (Willers 1926) that Captain Andreas Prytz made the first one by bending an iron rod twice and working one end into a pointer P and the other into a rockered knife edge K, as in figure 1a. Holding this giant staple upright at P, he moved P around the contour of an area with the knife edge zigzagging over the paper in pursuit. When he measured the *net* arc K_0K_1 between the start and finish positions of K, he found that its product with the much larger distance PK gave the area quite accurately.

This product can be interpreted as the area $K_0K_1P'P_0$ in figure 1 obtained by drawing the arc K_0K_1 about P_0 and the arc P_0P' about K_1, with K_1P' parallel to K_0P_0. In fact the area circumnavigated by P was proved by Willers (1926) using his general planimeter theorem (Willers 1948) to be exactly equal to the shaded area in figure 1, which is close to $K_0K_1P'P_0$.

However this identity does not help much in understanding how best to minimize or correct the inaccuracy of Prytz's approximate formula. For this Willers (1926) quotes the result of an analysis by Runge; the proof for a very similar infinite series by Hill (1894) is very laborious. More befitting the utter physical simplicity of the device is the following new analysis in terms of the rest of figure 1b.