Intrinsic Properties of Carbon Stars. I. Effective Temperature Scale of N-type Carbon Stars

Takashi Tsuji Tokyo Astronomical Observatory, University of Tokyo, Mitaka, Tokyo, 181 Japan

Received 1981 January 27; accepted 1981 March 7

Abstract. It is shown that the infrared flux method for determining stellar effective temperatures (Blackwell and Shallis 1977; Blackwell, Petford and Shallis 1980) can be applied to cool carbon stars. Although the spectra of cool carbon stars are highly line blanketed, the spectral region between 3 and 4 µm (L-band in the infrared photometry system) is found to be relatively free from strong line absorption. The ratio $R_L$ of bolometric flux to $L$ flux can then be used as a measure of effective temperature. On the basis of the predicted line-blanketed flux based on model atmospheres, with an empirical correction for the effect of 3 µm absorption due to polyatomic species (HCN, C$_2$H$_2$), it is shown that $R_L$ is roughly proportional to $T_{\text{eff}}^3$. The high sensitivity of $R_L$ to $T_{\text{eff}}$ makes it a very good measure of effective temperature, and the usual difficulty due to differential line blanketing effect in the analyses of photometric indices of cool carbon stars can be minimized.

It is found that the majority of N-type carbon stars with small variability (SRb and Lb variables) are confined to the effective temperature range between 2400 and 3200 K, in contrast to M-giant stars (M0 III- M6 III, including SRb and Lb variables) that are confined to the effective temperature range between 3200 and 3900 K. The effective temperatures based on the infrared flux method show good agreement with those derived directly from angular diameter measurements of 5 carbon stars. On the basis of the new effective temperature scale for carbon stars, it is shown that the well known C-classification does not represent a temperature sequence. On the other hand, colour temperatures based on various photometric indices all show good correlations with our derived effective temperatures.

Key words: carbon stars—effective temperatures—line blanketing—spectral classification—stellar atmospheres
Carbon stars play an important role in our understanding of both stars and galaxies. For example, the recent theory of stellar evolution made it possible to predict the details of stellar evolution at the advanced stages (e.g. Sugimoto and Nomoto 1974; Iben 1975). A carbon star could thus be a touchstone for the theory of stellar evolution at such late stages. Although carbon stars represent rather a minor constituent as compared with K–M giant stars in our Galaxy, it is found recently that carbon stars may form a more important population in other stellar systems such as the Magellanic Clouds (Blanco, Blanco and McCarthy 1978). This fact implies that carbon stars cannot necessarily be regarded as peculiar stars but rather they could represent a characteristic stage of normal stellar evolution. This fact also implies that carbon stars can be used as an important tracer of galactic evolution. Also, recent infrared and radio observations have revealed that many carbon stars are embedded in dust and molecular clouds (e.g. Merrill and Ridgway 1979; Zuckerman 1980), which implies that carbon stars are losing mass at a large rate and this has important bearings not only on stellar evolution but also on interstellar chemistry.

One important prerequisite to pursue these problems relating to carbon stars is to have an accurate knowledge of the intrinsic properties of carbon stars. This should in principle be determined from observations of nearby carbon stars. Unfortunately, this problem is still not adequately resolved possibly because of the great complexity of the spectra of cool carbon stars. For example, the effective temperature scale of carbon stars is not available. It is also uncertain whether any of the spectral classification schemes can be considered as a temperature sequence. Recently, spectral classification of carbon stars has been extended to a large sample by Yamashita (1967, 1972, 1975a), and major spectroscopic characteristics of these stars have been well worked out. On the other hand, Richer (1971) pointed out that C-classification is not compatible with colour temperatures based on infrared photometry, and he has proposed a new spectral classification scheme of carbon stars that may be more consistent with colour temperatures. Such an ambiguous situation can exist if either the C-classification does not represent the temperature sequence of carbon stars (e.g. Bergeat et al. 1976a; Scalo 1976), or if the interpretation of infrared photometry is incorrect because of the large differential line-blanketing effect in carbon stars (Yamashita 1975b). More recently, angular diameter measurements have also been made of some cool carbon stars (Ridgway, Wells and Joyce 1977; Ridgway, Jacoby, Joyce and Wells 1980, personal communication; Walker, Wild and Byrne 1979). The temperature scale of carbon stars can, in principle, be determined by this method. At present, however, the number of stars analyzed by this procedure is still too small to provide a definite conclusion. We show in this study a new possibility to settle the issue.

The infrared flux method for determining stellar effective temperature, first proposed by Blackwell and Shallis (1977) and further developed by Blackwell, Petford and Shallis (1980), can be applied to cool carbon stars. This success is due to locating an infrared spectral region that is free from strong line absorption in cool carbon stars. In fact, the $L$ band ($\lambda_{\text{eff}} = 3.4 \, \mu\text{m}$) measured by the current infrared photometric system can be regarded as a good quasi-continuum in cool carbon stars, and reasonably accurate prediction of the emergent stellar flux can be done on the basis of model
Intrinsic properties of carbon stars. I

97

atmospheres presently available for these stars. This fact makes it possible to apply the infrared flux method to these. Hence an effective temperature scale for the cool carbon stars can be determined reliably for the first time, by the same and consistent method as for K–M giant stars (Tsuji 1979, 1980).

2. Method of analysis

There have been two major methods for the determination of stellar effective temperatures. The first one utilizes the observed angular diameter $\theta_{LD}$ and the bolometric flux at the Earth $f_{\text{bol}}$. The effective temperature is obtained from

$$T_{\text{eff}} = (4f_{\text{bol}}/\sigma \theta_{LD})^{1/4}, \quad (1)$$

where $\sigma$ is the Stefan-Boltzman constant. This method has recently been applied successfully to cool stars on the basis of angular diameters of 5 carbon stars that have been measured by the lunar occultation method.

The second one is the model atmosphere method in which the temperature dependence of the observed spectral energy distribution curve is analyzed by means of model atmospheres. Although this method has been applied recently to cool stars, the spectral energy distribution of cool carbon stars is much more complicated as compared with that of M-giant stars. This spectral energy distribution is shown in Fig. 1 where the predicted line blanketed flux, based on the model of a carbon star with $T_{\text{eff}} = 3400$ K, $\log g = 0.0$ and $V_{\text{tur}} = 5$ km s$^{-1}$ (solid line, Querci and Tsuji 1974), is also shown together with the predicted line blocking-free continuum evaluated for the same model (dashed line). As compared with the emergent flux of an M-giant star of the same effective temperature (Tsuji 1978) shown at the bottom of Fig. 1 the emergent flux of a carbon star is much more disturbed by strong molecular absorption. In the case of M-giant stars, some quasi-continua such as those around 1 and 3.5 µm, are still found. It is then possible to determine effective temperatures for these stars by the second method since such quasi-continuous fluxes can be predicted accurately on the basis of model atmospheres. However, for carbon stars, such an analysis will be more difficult because it is not easy to find such quasi-continua in these stars. It is in principle possible to compute emergent fluxes for strongly line blanketed regions, but the accuracy of these computations may not be very high because of the high sensitivity of the results to the uncertainties in the model atmospheres as well as in the molecular data. In such a case, the determination of effective temperature will further be difficult because line absorption depends on other parameters such as chemical composition, gravity, turbulent velocity etc.

A new method of determining stellar effective temperature has been proposed recently by Blackwell and Shallis (1977). This third method, which is referred to as the infrared flux method, is based on the estimate of stellar angular diameter by

$$\theta_{LD} = [4f_{\lambda}/\pi F_{\lambda}(T_{\text{eff}}, g, \ldots)]^{1/8}, \quad (2)$$

where $F_{\lambda}$ is the observed infrared flux received at the Earth and $F_{\lambda}(T_{\text{eff}}, g, \ldots)$ is the emergent stellar flux. Then, with the bolometric flux, the effective temperature
Figure 1. Spectral energy distribution of carbon stars (top) is shown in comparison with that of M-giant stars (bottom) of the same effective temperature. Predicted line blanketed flux is shown by solid line while line blocking free continuum based for the same model atmosphere is shown by dashed line in each model. It is to be noted that the spectral region between 3 and 4 $\mu$m is little disturbed by strong line blanketing even in a carbon star.

can be determined by equation (1). The final solution in this method should be determined by iterations since the emergent stellar flux in equation (2) already depends on the effective temperature and other stellar parameters. Recently, however, Blackwell, Petford and Shallis (1980) showed that the iterations can be avoided eliminating $\theta_{LD}$ from equations (1) and (2). Thus we derive

$$f_{\text{bol}}/f_\lambda = T_{\text{eff}}^4/\pi R_\lambda (T_{\text{eff}}, g, \ldots) = R_\lambda (T_{\text{eff}}, g, \ldots).$$

Then, the observed value of $f_{\text{bol}}/f_\lambda$ can be interpreted in terms of $R_\lambda (T_{\text{eff}}, g, \ldots)$ that can be evaluated on the basis of model atmospheres, and the effective temperature can be estimated if the values of other parameters are known (or are not important). Also, infrared flux $f_\lambda$ depends linearly on $T_{\text{eff}}$ in the Rayleigh-Jeans region, and hence $R_\lambda$ varies as $T_{\text{eff}}$. Because of this high sensitivity of $R_\lambda$ to $T_{\text{eff}}$, $R_\lambda$ can be used as a good stellar thermometer.

It is also desirable to apply the infrared flux method to the spectral region least disturbed by line absorption. In this method, however, it is necessary to find only one spectral region that is free from strong line absorption in the infrared. This is an important advantage of the method especially for carbon stars, because it is possible
to find a spectral region little disturbed by strong absorption even in cool carbon stars. In fact, the difference between the line blanketed and the line free fluxes is small in the spectral interval between 3 and 4 μm as seen in Fig. 1. This fact suggests that the $L$ flux of carbon star is also a good quasi-continuum. Also, detailed spectrophotometric observations of this interval (e.g. Merrill and Stein 1976; Noguchi et al. 1977; Goebel et al. 1980) show that there is no strong absorption in the $L$ band region except for the characteristic absorption centred at 3·07 μm. The spectral energy curve beyond this absorption is pretty smooth up to 4 μm especially in the non-Mira carbon stars. This fact implies that the emergent flux for $L$ band can be predicted with reasonable accuracy even for cool carbon stars and thereby opens a new possibility of applying the infrared flux method to cool carbon stars.

3. Emergent fluxes and $R$ values for carbon stars

Our problem is to evaluate accurate $L$ flux and corresponding $R_L$ value for carbon stars. Until now line blanketed fluxes for carbon stars have been determined by the opacity probability distribution function method (OPDF, Querci Querci and Tsuji 1974; Querci and Querci 1976) or by the opacity sampling method (Sneden, Johnson and Krupp 1976). A problem in model atmospheres for carbon stars concerns the assumption of a composition especially of C, N and O. One hypothesis is that the atmospheres of carbon stars are mainly composed of the material processed through CNO cycle, and the emergent fluxes based on this idea have been evaluated by Querci, Querci and Tsuji (1974). This type of model atmospheres is characterised by a high abundance of nitrogen and we refer to this series of models as N-rich models. Another hypothesis is that the origin of carbon stars should be due to mixing of material processed through 3α reaction, and some emergent fluxes for such models have been given by Querci and Querci (1976). These atmospheres are carbon-rich and we refer to this series of models as C-rich models.

Another problem is a possible effect of 3 μm absorption which is situated at the edge of the $L$ band filter. The 3 μm absorption is now known to be due to polyatomic molecules such as C$_2$H$_2$ and HCN (Ridgway, Hall and Carbon 1978). In the model atmospheres of carbon stars noted above, the effect of these polyatomic molecules has not yet been taken into account. For our present purpose, however, we find that an empirical approach based on the observed band absorption may be sufficient. According to a detailed study of 3 μm absorption in carbon stars by Noguchi et al. (1977), the observed flux $F_{\text{obs}} (\lambda)$ around 3 μm band can be represented by

$$F_{\text{obs}} (\lambda) = F_0 (\lambda) \exp [-\tau (\lambda)], \quad (4)$$

where $\tau (\lambda)$ is the optical depth of the 3 μm band and $F_0(\lambda)$ is the local pseudo continuum around the 3 μm band. These authors have further shown that

$$\tau_{\text{norm}} (\lambda) = \tau(\lambda) / \tau_m \quad (5)$$

normalized by the maximum optical depth $\tau_m$ is remarkably similar for all the carbon stars observed, both in the position of the maximum depth and in the band shape (see Figs 11-13 of Noguchi et al. 1977).
The problem now is to assign the value of $\tau_m$ for a model of a given effective temperature. For this purpose, we find that the $3 \mu m$ band optical depths $\tau_m$ measured by Noguchi et al. (1977) correlate very well with the effective temperatures determined by the infrared flux method. This correlation was already found for the effective temperatures determined by $R_L$ values which have been evaluated without the effect of $3 \mu m$ absorption. With this correlation between $\tau_m$ and $\tau_{\text{eff}}$, it is now possible to assign a value of $\tau_m$ for a model of a given effective temperature. Assuming that the normalized band shape $\tau_{\text{norm}}$ represented by a triangular dip centred at $3.07 \mu m$ (see Fig. 17 of Noguchi et al. 1977), the effect of $3 \mu m$ band on the emergent flux can be evaluated by equation (4) and (5). The resulting $L$ flux and $R_L$ values can be used to estimate the improved values of effective temperatures. After a few iterations, the values finally found are $\tau_m = 0.15, 0.3$ and $0.9$ for $T_{\text{eff}} = 3400, 3000$ and $2600$ K respectively. More detailed discussion on this correlation will be given in a subsequent paper (Paper II: Tsuji 1981). In Fig. 2, the solid lines are the predicted line blanketed fluxes based on OPDF opacity for CN, C$_2$ and CO (Querci, Querci and Tsuji 1974). As the absorption lines due to diatomic molecules are rather uniformly distributed in this spectral region, these line blanketed fluxes can be identified with the pseudo continuum $F_0(\lambda)$ defined in equation (4). Against this pseudo

![Figure 2](image-url)  

**Figure 2** Spectral energy distributions between 3 and 4 $\mu m$ are shown for models with $T_{\text{eff}} = 3400$, 3000 and 2600 K. The solid lines represent predicted line blanketed fluxes including the effect of diatomic molecules such as CN, CO and C$_2$, while dashed lines further include the effect of polyatomic molecules, mostly of HCN and C$_2$H$_2$, estimated by empirical method outlined in the text.
Intrinsic properties of carbon stars. I

continuum, the dashed line represents the polyatomic absorption estimated for each model with the corresponding values of \( \tau_m \).

Based on the emergent fluxes such as those shown in Fig. 2, integrated \( L \) fluxes are evaluated by

\[
F_L = \int F_\lambda S_\lambda d\lambda / \int S_\lambda d\lambda,
\]

where \( S_\lambda \) is the response function of the \( L \) filter (Johnson 1965). Then, \( R_L \) values are obtained from

\[
R_L = \sigma T_{eff}^4 / \pi F_L.
\]

The resulting \( L \) fluxes and \( R_L \) values are given in Table 1, for the cases with and without 3 \( \mu \)m absorption. As the effect of gravity is found to be very small (the difference in \( \log F_L \) is less than 0.01 for a difference of \( \log g \) by 1.0), the mean value for models with different gravities is given for each effective temperature. The effect of 3 \( \mu \)m absorption is \( \Delta \log R = 0.020, 0.031 \) and 0.084 for \( T_{eff} = 3400, 3000 \) and 2600 K respectively. These differences in predicted \( R_L \) values give differences of \( \Delta T_{eff} = 20, 30 \) and 200 K in resulting effective temperatures at around \( T_{eff} = 3400, 3000 \) and 2600 K respectively. Thus, the effect of 3 \( \mu \)m absorption is not very important, except for the coolest stars. This is as expected since the 3 \( \mu \)m absorption is situated where the response function of the \( L \) filter is already rapidly decreasing. The effect of chemical composition is found to be of some importance only for the coolest models. In Fig. 3, values of \( \log R_L \) (with the effect of 3 \( \mu \)m absorption included) for C-rich and N-rich series are shown by solid and dashed lines, respectively. The effective temperatures, determined from a given value of observed \( \log R \) using the predicted \( \log R_L - T_{eff} \) relations shown in Fig. 3 may differ by about 200 K for C-rich and N-rich cases at around \( T_{eff} = 2600 \) K, but the difference may be less than 100 K in most of the cases with \( T_{eff} \) above 2800 K.

Table 1. Predicted \( L \) fluxes and \( R \) values for carbon stars.

<table>
<thead>
<tr>
<th>Model atmospheres</th>
<th>Line blanketed flux</th>
<th>Line blanketed flux²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series</td>
<td>( T_{eff} )</td>
<td>( \log g )</td>
</tr>
<tr>
<td>N</td>
<td>4500</td>
<td>( -1.0 \sim 1.0 )</td>
</tr>
<tr>
<td>N</td>
<td>4200</td>
<td>( -1.0 \sim 1.0 )</td>
</tr>
<tr>
<td>N</td>
<td>3800</td>
<td>( -1.0 \sim 1.0 )</td>
</tr>
<tr>
<td>N</td>
<td>3400</td>
<td>( -1.0 \sim 1.0 )</td>
</tr>
<tr>
<td>N</td>
<td>3000</td>
<td>0.0</td>
</tr>
<tr>
<td>N</td>
<td>2600</td>
<td>1.0</td>
</tr>
<tr>
<td>C</td>
<td>3600</td>
<td>1.0</td>
</tr>
<tr>
<td>C</td>
<td>3000</td>
<td>0.0</td>
</tr>
<tr>
<td>C</td>
<td>2600</td>
<td>1.0</td>
</tr>
</tbody>
</table>

¹Line blanketed fluxes based on the OPDF opacities for C\(_2\), CN and CO; N-rich models by Querci, Querci and Tsuji (1974) and C-rich models by Querci and Querci (1976).

²With further empirical correction for the effect of 3 \( \mu \)m absorption; see Section 4 for details.
**Figure 3.** The values of $\log R_L$, where $R_L$ is the ratio of bolometric flux to $L$ flux, are plotted against effective temperatures for C-rich models (solid line) and for N-rich models (dashed line). The effect of polyatomic absorption at 3.07 $\mu$m is taken into account in both the cases.

### 4. Effective temperatures of cool carbon stars

A problem one encounters in applying the infrared flux method to carbon stars is that most of these stars are variable and this fact makes it difficult to obtain a consistent set of photometric data. This difficulty, however, may not be very serious if we restrict our attention to Lb and SRb variables as defined by Kukarkin et al. (1969). Some characteristics of these variables were summarized by Peery (1975) who denoted these variables as N irregular variables. These variables show much smaller variabilities (typically 0.2 mag versus 1-2 mag at 1.04 $\mu$m) and smaller infrared excess (1 mag versus 1.5-2.5 mag at 11 $\mu$m), as compared with SRa and Mira variables.

The first extensive infrared photometry of carbon stars has been done by Mendoza and Johnson (1965). Further infrared flux measurements have occasionally been done by several authors and these results are summarized by Bergeat et al. (1976a) together
Intrinsic properties of carbon stars. I

with their own observations. The spectral energy distributions given by these authors are integrated to obtain bolometric fluxes. The interstellar reddenings for these relatively bright carbon stars are generally unimportant, but they are estimated by applying Parenago's formula (Sharov 1964) or else by applying the results of FitzGerald (1968). The distances are estimated with the assumption of $M_{1.04\mu m} = -4.3$ (Baumert 1974). The reddening law by Lee (1970) is applied. More recently, Walker (1980) has carried out new infrared photometry of southern carbon stars; he has also given bolometric magnitudes corrected for interstellar reddening.

Bolometric fluxes and $L$ fluxes both corrected for the effect of interstellar reddening, compiled from these sources are summarized in Table 2. On the basis of these data, observational values of $\log R_L$ can be computed and the effective temperatures can be determined from Fig. 3. The predicted $\log R_L - T_{\text{eff}}$ relation used for determining effective temperatures is based on the model atmospheres of C-series as defined in Section 3. In fact, the recent studies (e.g. Kilston 1975; Querci and Querci 1976; Thompson 1977) seem to support the hypothesis that the origin of carbon stars may be due to the mixing of carbon produced by He-burning rather than by H-burning in CNO cycle. The resulting effective temperatures are also given in Table 2. For a few stars photometric data from different sources do not agree. These differences may be mostly due to variabilities of these stars. The difference in the resulting effective temperatures are generally less than 150 K, which is the estimated accuracy of our analysis as will be discussed here. Inspection of the result reveals that the effective temperatures of N-type carbon stars are generally very low, mostly below 3200 K (only exception is BL Ori with $T_{\text{eff}} = 3420$ K, which should be further examined). This is in contrast to the effective temperatures of non-Mira M-giant stars that are generally hotter than 3200 K (e.g. Tsuji 1978; Ridgway et al. 1980).

The accuracy of the resulting effective temperatures depends on the accuracies of both observed and predicted $R_L$ values. As to the observed $R_L$ values, a major source of error may be the present uncertainty in absolute calibration of the infrared photometric system. Our analysis is based on the calibration by Johnson (1966) and its accuracy is estimated to be about 10 per cent. Even with the possible error of 10 per cent in observed $R_L$ values the corresponding error in resulting $T_{\text{eff}}$ is about 3 per cent (100 K) because of the high sensitivity of $R_L$ to $T_{\text{eff}}$ as noted before. On the other hand, the error due to the predicted $R_L$ values may be of the same order as those of the observed $R_L$ values in most cases, since the differences in the predicted $R_L$ values due to different assumptions on composition, gravity, line blanketing and so on are rather small so that the resulting uncertainties in $T_{\text{eff}}$ are generally less than 100 K as is shown in Section 3. Only in the coolest models of $T_{\text{eff}}$ near 2600 K, $R_L$ values are more sensitive to these assumptions and the possible error may be as large as 200 K. Considering these uncertainties both in observed and predicted $R_L$ values, the overall accuracy of our effective temperatures may be about 150 K for most cases.

Another problem to be examined is whether circumstellar dust emission has any importance to our analysis. For example, Bergeat et al. (1976 a, b, c) have employed a model in which dust thermal emission is of considerable importance in explaining the spectral energy distribution of carbon stars in the near infrared. They have applied such a model not only to Mira-types but also to non-Mira types covered in
Table 2. Spectral types, $L$ fluxes, integrated fluxes, and effective temperatures of N-type carbon stars.

<table>
<thead>
<tr>
<th>GCCCS</th>
<th>Name</th>
<th>Spectral types</th>
<th>$E_{B-V}$</th>
<th>log $f_L$</th>
<th>$f_{bol}$</th>
<th>log $R_L$</th>
<th>$T_{eff}$</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>T Cae</td>
<td>N4 C6, 3 C5 II</td>
<td>0.03</td>
<td>-3.868</td>
<td>-6.741</td>
<td>-2.873</td>
<td>3000 W</td>
<td>W</td>
</tr>
<tr>
<td>284</td>
<td>W Ori</td>
<td>N5 C5, 4 C6 II</td>
<td>0.06</td>
<td>-2.744</td>
<td>-5.749</td>
<td>-3.005</td>
<td>2620 W</td>
<td>W</td>
</tr>
<tr>
<td>297</td>
<td>SY Eri</td>
<td>N0 C6, 3 C5 II</td>
<td>0.08</td>
<td>-3.844</td>
<td>-6.777</td>
<td>-2.933</td>
<td>2830 B</td>
<td>B</td>
</tr>
<tr>
<td>508</td>
<td>BL Ori</td>
<td>Nb C6, 3 C5 II</td>
<td>0.03</td>
<td>-3.372</td>
<td>-6.093</td>
<td>-2.721</td>
<td>3420 B</td>
<td>B</td>
</tr>
<tr>
<td>537</td>
<td>UU Aur</td>
<td>N3 C6, 4 C5 II</td>
<td>0.04</td>
<td>-2.612</td>
<td>-5.546</td>
<td>-2.934</td>
<td>2825 B</td>
<td>B</td>
</tr>
<tr>
<td>1338</td>
<td>X Cnc</td>
<td>N3 C5, 4 C6 II</td>
<td>0.08</td>
<td>-3.036</td>
<td>-6.049</td>
<td>-3.013</td>
<td>2600 W</td>
<td>W</td>
</tr>
<tr>
<td>1714</td>
<td>U Hya</td>
<td>N2 C6, 3 C5 II</td>
<td>0.01</td>
<td>-2.584</td>
<td>-5.517</td>
<td>-2.933</td>
<td>2825 B</td>
<td>B</td>
</tr>
<tr>
<td>1736</td>
<td>VV UMa</td>
<td>N0 C6, 3 C5</td>
<td>0.02</td>
<td>-3.061</td>
<td>-5.984</td>
<td>-2.923</td>
<td>2855 B</td>
<td>B</td>
</tr>
<tr>
<td>2030</td>
<td>Y CVn</td>
<td>N3 C5, 5 J C7 I</td>
<td>0.01</td>
<td>-2.526</td>
<td>-5.493</td>
<td>-2.967</td>
<td>2730 B</td>
<td>B</td>
</tr>
<tr>
<td>2047</td>
<td>RY Dra</td>
<td>N4 C4, 5 J C7 I</td>
<td>0.02</td>
<td>-2.852</td>
<td>-5.900</td>
<td>-3.048</td>
<td>2500 B</td>
<td>B</td>
</tr>
<tr>
<td>2157</td>
<td>T Lup</td>
<td>N4 C4, 3 J C5 II</td>
<td>0.4</td>
<td>-3.676</td>
<td>-6.613</td>
<td>-2.938</td>
<td>2810 W</td>
<td>W</td>
</tr>
<tr>
<td>2160</td>
<td>RS Lup</td>
<td>N4 C4, 3 J C5 II</td>
<td>0.4</td>
<td>-3.676</td>
<td>-6.613</td>
<td>-2.938</td>
<td>2810 W</td>
<td>W</td>
</tr>
<tr>
<td>2173</td>
<td>Z Lup</td>
<td>Na C4, 3 J C5 II</td>
<td>0.18</td>
<td>-3.684</td>
<td>-6.633</td>
<td>-2.949</td>
<td>2780 W</td>
<td>W</td>
</tr>
<tr>
<td>2219</td>
<td>X Tra</td>
<td>C5 II</td>
<td>0.06</td>
<td>-2.616</td>
<td>-5.601</td>
<td>-2.985</td>
<td>2680 W</td>
<td>W</td>
</tr>
<tr>
<td>2240</td>
<td>U Aps</td>
<td>C5 II</td>
<td>0.16</td>
<td>-3.548</td>
<td>-6.545</td>
<td>-2.997</td>
<td>2640 W</td>
<td>W</td>
</tr>
<tr>
<td>2362</td>
<td>V Tra</td>
<td>N4 C4, 3 J C5 II</td>
<td>0.18</td>
<td>-4.016</td>
<td>-6.837</td>
<td>-2.821</td>
<td>3140 W</td>
<td>W</td>
</tr>
<tr>
<td>2478</td>
<td>SZ Sgr</td>
<td>Nb C7, 3 J C5 II</td>
<td>0.21</td>
<td>-3.780</td>
<td>-6.625</td>
<td>-2.845</td>
<td>3080 W</td>
<td>W</td>
</tr>
<tr>
<td>2499</td>
<td>V781 Sgr</td>
<td>N4 C5, 5 J C5 II</td>
<td>0.8</td>
<td>-3.420</td>
<td>-6.369</td>
<td>-2.949</td>
<td>2780 W</td>
<td>W</td>
</tr>
<tr>
<td>2608</td>
<td>T Lyr</td>
<td>N4 C5, 5 J C5</td>
<td>0.06</td>
<td>-2.925</td>
<td>-6.013</td>
<td>-3.088</td>
<td>2380 B</td>
<td>B</td>
</tr>
<tr>
<td>2620</td>
<td>RX Sct</td>
<td>N3 C5, 5 J C5 II</td>
<td>0.8</td>
<td>-3.488</td>
<td>-6.341</td>
<td>-2.853</td>
<td>3055 W</td>
<td>W</td>
</tr>
<tr>
<td>2660</td>
<td>S Sco</td>
<td>N3 C5, 5 J C5 II</td>
<td>0.4</td>
<td>-3.092</td>
<td>-6.001</td>
<td>-2.909</td>
<td>2895 W</td>
<td>W</td>
</tr>
<tr>
<td>2695</td>
<td>V Aql</td>
<td>N6 C5, 5 J C5 II</td>
<td>0.25</td>
<td>-2.828</td>
<td>-5.837</td>
<td>-3.009</td>
<td>2610 W</td>
<td>W</td>
</tr>
<tr>
<td>2721</td>
<td>V1942 Sgr</td>
<td>N6 C6, 4 J C5 II</td>
<td>0.15</td>
<td>-3.320</td>
<td>-6.229</td>
<td>-2.909</td>
<td>2895 W</td>
<td>W</td>
</tr>
<tr>
<td>2744</td>
<td>AQ Sgr</td>
<td>N4 C5, 5 J C5 II</td>
<td>0.15</td>
<td>-3.208</td>
<td>-6.210</td>
<td>-3.002</td>
<td>2630 W</td>
<td>W</td>
</tr>
<tr>
<td>2785</td>
<td>UW Sgr</td>
<td>C3 II</td>
<td>0.2</td>
<td>-4.340</td>
<td>-7.169</td>
<td>-2.829</td>
<td>3120 W</td>
<td>W</td>
</tr>
<tr>
<td>2882</td>
<td>RT Cap</td>
<td>N3 C6, 4 J C6 II</td>
<td>0.10</td>
<td>-3.104</td>
<td>-6.109</td>
<td>-3.005</td>
<td>2620 W</td>
<td>W</td>
</tr>
<tr>
<td>3013</td>
<td>T Ind</td>
<td>Na C7, 2 J C5 II</td>
<td>0.03</td>
<td>-3.204</td>
<td>-6.065</td>
<td>-2.861</td>
<td>3030 W</td>
<td>W</td>
</tr>
<tr>
<td>3018</td>
<td>Y Pav</td>
<td>N4 C6, 4 J C5 II</td>
<td>0.07</td>
<td>-3.056</td>
<td>-5.921</td>
<td>-2.865</td>
<td>3020 W</td>
<td>W</td>
</tr>
<tr>
<td>3060</td>
<td>V460 Cyg</td>
<td>N1 C6, 3 J C5 II</td>
<td>0.06</td>
<td>-2.977</td>
<td>-5.903</td>
<td>-2.926</td>
<td>2845 B</td>
<td>B</td>
</tr>
<tr>
<td>3061</td>
<td>RR Ind</td>
<td>N1 C6, 3 J C5 II</td>
<td>0.07</td>
<td>-3.416</td>
<td>-7.173</td>
<td>-2.857</td>
<td>3040 W</td>
<td>W</td>
</tr>
<tr>
<td>3202</td>
<td>TX Psc</td>
<td>N0 C7, 2 J C5 II</td>
<td>0.00</td>
<td>-2.684</td>
<td>-5.577</td>
<td>-2.893</td>
<td>2945 W</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
<td>-2.735</td>
<td>-5.583</td>
<td>-2.848</td>
<td>3070 B</td>
<td>B</td>
</tr>
</tbody>
</table>

Notes:
(1) Stephenson (1973).
(2) N-classification (Shane 1928; Yamashita 1972, 1975a).
(3) C-classification (Keenan and Morgan 1941; Yamashita 1972, 1975a).
(4) Richer's (1971) classification.
(5) From Walker (1980) for his sample, and estimations by the method outlined in the text for other stars.
(6) $f_L$ in unit of erg cm$^{-3}$ per $\Delta \lambda = 1$ cm.
(7) $f_{bol}$ in unit of erg cm$^{-2}$.
(8) Major sources of photometric data; B:Bergeat et al. (1976a) and references cited therein, W: Walker (1980).
the present study. Recently, however, Walker (1980) has concluded from his analysis of the two-colour diagram that there is no evidence of dust thermal emission in the near infrared for most N-type irregulars. Also, detailed photometric analysis of the infrared spectra of Υ CVn that extends to 30 µm and for which Bergeat et al. (1976a, b, c) suggest possible importance of dust shell, reveal almost no contribution of dust thermal emission (Goebel et al. 1980). Furthermore, the very fact that the 3 µm absorption is very deep in some non-Mira N-type stars (Noguchi et al. 1977) clearly indicates that the dust thermal emission will be of little importance in the near infrared at least around the L band. For these reasons, we conclude that the effect of dust is not important to our analysis for the sample of stars covered in the present study. The effect of dust thermal emission, however, may be of importance in Mira-type carbon stars (e.g. Merrill and Stein 1976). In fact, this possibility together with the large variability and very heavy line blanketing effect in these stars make it rather difficult to apply the infrared flux method.

5. Comparisons with other temperature scales

Until recently several attempts were made to determine the temperature scale for carbon stars. The most direct method was to determine the effective temperatures from the angular diameters directly measured. Recently, angular diameter measurements by the lunar occupation method have been successfully applied to 5 carbon stars (Ridgway, Wells and Joyce 1977; Ridgway, Jacoby, Joyce and Wells 1980, personal communication; Walker, Wild and Byrne 1979), of which 4 carbon stars (TX Psc, X Cnc, AQ Sgr, SZ Sgr) are N-irregulars while one (Y Tau) is Sra variable (Table 3). For these five stars, effective temperatures and angular

<table>
<thead>
<tr>
<th>Star</th>
<th>Lunar occultation method</th>
<th>Infrared flux method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>θ_{LD} (milliarcsec)</td>
<td>T_{eff} (K)</td>
</tr>
<tr>
<td>TX Psc</td>
<td>9.31 ± 0.75^{(1)}</td>
<td>3080 ± 150^{(1)}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3114 ± 150^{(2)}</td>
</tr>
<tr>
<td>X Cnc</td>
<td>8.79 ± 1.00^{(3)}</td>
<td>2491 ± 138^{(1)}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2385 ± 140^{(3)}</td>
</tr>
<tr>
<td>AQ Sgr</td>
<td>5.97 ± 0.49^{(2)}</td>
<td>2684 ± 110^{(3)}</td>
</tr>
<tr>
<td>SZ Sgr</td>
<td>3.21 ± 0.16^{(3)}</td>
<td>2776 ± 70^{(3)}</td>
</tr>
<tr>
<td>Y Tau</td>
<td>8.58 ± 0.40^{(1)}</td>
<td>2565 ± 70^{(3)}</td>
</tr>
</tbody>
</table>

Notes:
(1) Ridgway, Wells and Joyce (1977).
(3) Ridgway, Jacoby, Joyce and Well (1980), personal communication.
(4) Photometric angular diameter by equation (2) with the adopted effective temperature.
(5) From Table 2 except for Y Tau. For Y Tau, photometric data by Bergeat et al. (1976a) are used after correction of interstellar reddening with E_{B-V} = 0.06, log f_L = -2.954, log f_{bol} = -5.993, and log R_L = -3.039.
diameters by the lunar occupation method and by the infrared flux method (Table 2) are compared in Fig. 4. Agreements between the effective temperatures by the two methods are remarkably good for TX Psc, AQ Sgr and Y Tau (this star is SRa variable and therefore not included in our sample; for purposes of comparison, we tentatively apply the infrared flux method to it with the photometric data noted in Table 3). The effective temperatures by the two methods show positive correlation and the mean difference is $\Delta T_{\text{eff}} = -69 \text{ K} \pm 82 \text{ K}$ (mean error) for 5 stars, where $\Delta T_{\text{eff}}$ is $T_{\text{eff}}$ (angular diameter method) $- T_{\text{eff}}$ (infrared flux method). Thus, the infrared flux method as applied to cool carbon stars results in effective temperatures consistent with those by the lunar occupation method. However in the case of X Cnc and SZ Sgr the two methods give a temperature difference of about 10 per cent.

Other temperature scales for carbon stars have mostly been based on the infrared colour indices. In Fig. 5a, b, c and d, temperatures estimated by Mendoza and Johnson (1965), by Richer (1971), by Bergeat et al. (1976c), and by Walker (1980), respectively, are plotted against our effective temperatures. Mendoza and Johnson (1965) estimated their effective temperatures by applying the colour index—effective temperature calibration for K-M giant stars to carbon stars. This procedure, however, cannot be justified for two reasons; firstly the photometric properties of carbon

Figure 4. Comparison of the effective temperatures determined by the infrared flux method (abscissa) are compared with those by the angular diameters directly measured by the lunar occultation method (ordinate). Mean values are used when two estimates are available in Table 3. The dashed line corresponds to $T_{\text{eff}}$ (AD) = $T_{\text{eff}}$ (IR) while the solid line corresponds to the mean difference of $T_{\text{eff}}$(AD) $-T_{\text{eff}}$ (IR) = $-69 \text{ K}$. 
stars may be different from those of K-M giant stars (Paper II) and, secondly the calibration for KM giant stars which they have applied is now known to be erroneous by several hundred degrees (e.g. Tsuji 1978; Ridgway et al. 1980). For these reasons, it is possible that the apparently good agreement in effective temperature values between those obtained by Mendoza and Johnson (1965) and ours may be fortuitous. Other temperature determinations were based on the calibrations of infrared colour indices by means of blackbodies. Although some authors suggest that carbon stars radiate roughly like blackbodies and hence the colour temperatures can be good approximations to effective temperatures (e.g. Scalo 1976), it is difficult to justify this in view of the large line blanketing effect in carbon stars (see Fig. 1). In fact, the results of different authors which are based on different colour indices show systematic differences with our effective temperatures in different ways, as shown in Fig. 5. Also, the results of Bergeat et al. (1976) are based on the assumption that dust thermal emission is of some importance, but this may probably be an overestimate as mentioned in Section 4. One important conclusion from Fig. 5, is that all the colour-temperatures show positive correlation with our effective temperatures, and that they can be used as measures of effective temperatures if properly calibrated.

Figure 5. Colour temperatures estimated by several authors are compared with the effective temperatures based on the infrared flux method; (a) Mendoza and Johnson (1965), (b) Richer (1971), (c) Bergeat et al. (1976), (d) Walker (1980). One deviating star in (a) and (b) is RX Sct for which we assume large reddening of $E_{B-V} = 0.8$, following Walker (1980), while other authors probably did not. For this reason, this star can be omitted from comparisons. In fact, this star is also included in (d) and it shows no deviation from the mean relation, because the same reddening is used.
6. Spectral classification of cool carbon stars

As compared with the normal red giant stars for which MK classification can be applied, spectral classification of carbon stars has been more difficult (Fujita 1980). Cool carbon stars have been classified as N-type in the Harvard classification, and Shane (1928) had later revised the sub-classification of N-type on the basis of the spectral gradient in short wavelengths (violet and blue). In the upper panel of Fig. 6, N-subtypes due to several authors, as summarized by Yamashita (1972, 1975a), are plotted against the effective temperatures we have derived. This plot reveals that the dependence of N-subtypes on effective temperatures is not strong. It was, however,

Figure 6. Spectral types are plotted against effective temperatures; (a) R -N classification (Shane 1928) (b) C-classification (Keenan and Morgan 1941; Yamashita 1972), (c) Richer's classification (1971)
Intrinsic properties of carbon stars.

noted by Shane (1928) himself that the redness of carbon stars cannot be satisfactorily related to stellar temperatures because colour temperatures based on violet-blue fluxes were too low (below 1000 K for some stars) and that some selective absorption may play an important role.

Keenan and Morgan (1941) have proposed a new classification scheme of carbon stars based on several spectroscopic features that may be sensitive to temperature. This classification is now known as the C-classification and is most commonly used in studies of carbon stars. In the middle of Fig. 6, temperature classes of C-classification by Yamashita (1972, 1975a) are plotted against our effective temperatures. Although the scatter in effective temperatures for a given spectral type is rather large, the general trend between C4 and C7 is opposite to what one might expect from the original intent of the spectral classification. This conclusion can be extended to C-classifications done by other authors, since Yamashita's classification shows good agreement with those by Keenan and Morgan (1941), by Bouigue (1954), and by Yamashita (1967) as has been shown by Yamashita (1972).

Recently, another classification scheme has been proposed by Richer (1971) on the basis of the near infrared spectra of carbon stars. In the lower panel of Fig. 6, Richer's temperature classes are plotted against our effective temperatures. Again, the dispersion in effective temperatures at a given spectral type is rather large, but this spectral classification shows better correlation with effective temperatures, as compared with the other classifications. It is also consistent with the fact that Richer's classification shows an inverse correlation with C-classification, as has been noted by Yamashita (1972).

Until recently, some questions on the validity of the C-classification as a temperature sequence have been expressed already by several authors. Especially, it was pointed out that temperatures determined from infrared colour indices generally show poor correlation, or even anti-correlation, with C-classification (e.g. Richer 1971; Scalo 1976; Bergeat et al. 1976c). However, the temperatures determined by these authors are essentially colour temperatures as has been noted in Section 5. Differential line blanketing effect by molecular bands or of interstellar reddening can complicate the problem. The validity of the C-classification as a temperature classification can still be defended, as discussed in detail by Yamashita (1975b). It is to be remembered that the values of effective temperatures we derive are almost free from the differential line blanketing effect. As shown in Section 5 the colour temperatures show a good correlation with our effective temperatures.

The possibility that the later C-type stars may have higher effective temperatures than the earlier C-type stars was also suggested on the basis of effective temperatures derived from angular diameters measured by the lunar occupation method for three stars (Ridgway, Wells and Joyce 1977). Although the number of stars covered by their analysis was too small, our analysis is now extended to a larger sample of stars so that a general conclusion can be made. Thus, we can finally confirm that it is the C-classification that is incorrect as a temperature indicator for cool carbon stars, at least between C4 and C7.

7. Discussion and conclusions

We have determined effective temperatures for a large sample of cool carbon stars
on the basis of the infrared flux method developed by Blackwell, Petford and Shallis (1980). Our analysis reveals that the majority of N-type carbon stars, which are also classified as SRb or Lb variables, are confined to the effective temperature range between 2400 and 3200 K. In contrast, M-giant stars of the spectral type between M0 and M6, the latter half of which are mostly variables of SRb and Lb types (Kukarkin et al. 1969), are confined to the effective temperature range between 3200 and 3900 K (Tsuji 1978, 1980; Ridgway et al. 1980). Thus, red variables of small amplitude (SRb and Lb types) are sharply divided at about $T_{\text{eff}} \sim 3200$ K into oxygen-rich and carbon-rich giants. It is to be noted that Mira and SRa type variables are not included in our sample both for M giants and carbon stars. To extend the temperature calibration to these large amplitude variables should be a major problem to be studied in the future.

Because of the well known complexity of the spectra of cool carbon stars and, especially because of the heavy line blanketing effect in these stars, it was generally not expected that the effective temperatures of cool carbon stars can be determined by the analysis of the photometric data. The present author also expressed rather a pessimistic view that the effective temperature scale of carbon stars can be determined only if direct measurements of angular diameters can be extended to a large number of carbon stars in future (Tsuji 1979). By the infrared flux method, however, it is shown that the difficulty due to heavy line blanketing effect can be avoided to some extent, because this method utilizes the ratio of the integrated flux which is free from differential line blanketing effect, and one infrared flux that can be chosen to be free from strong line absorption. In fact, it can be confirmed that this expectation is indeed realized by the following lines of evidence. Our effective temperatures show good agreement with those based on the angular diameters measured by the lunar occultation method (Ridgway, Wells and Joyce 1977; Ridgway, Jacoby, Joyce and Wells 1980, personal communication; Walker, Wild and Byrne 1979) as is shown in Fig. 4, and colour temperatures defined by various photometric indices all show good correlations with our effective temperatures (Fig. 5). The spectroscopic as well as photometric properties of cool carbon stars can most consistently be understood on the basis of the effective temperatures we desire. We propose to show this in detail in Paper II.

Such a success in applying the infrared flux method to cooler stars with complicated spectra like carbon stars demonstrates that the infrared flux method is a truly fundamental technique in stellar astronomy. In fact, this method has already been applied not only to temperature determinations of normal stars (e.g. Blackwell Shallis and Selby 1979; Blackwell, Petford and Shallis 1980) but also to some peculiar stars (Shallis and Blackwell 1979) and also to the calibration of stellar radii (Shallis and Blackwell 1980). Furthermore, an important advantage of this method is its simplicity; although our analysis is presently limited to 31 carbon stars, this method can be easily applied to any individual carbon star for which the effective temperature is required. For this purpose, what is necessary is a consistent set of photometric data covering the near infrared (including the L band).

The accuracy of our values of effective temperature is estimated to be about 150 K for most carbon stars. For improvement in accuracy, accurate calibration of the infrared photometry is important. Recently an attempt of direct comparison between the stellar flux and a blackbody source at $K$-band has been done by Selby et al. (1980),
and extension of this work to other bands, now in progress at Tenerife (Blackwell 1980), should provide more accurate calibration in the near future. The accuracy of the resulting effective temperatures also depends on the accuracy of the predicted $R_E$ values and, for this reason, the infrared flux method is model dependent. The present limitations on the model atmospheres of carbon stars have been reviewed, for example, by Carbon (1979). Especially, in view of very low effective temperatures of carbon stars, the effect of polyatomic molecular opacity should be more important in carbon stars than in M-giant stars. Certainly, model atmospheres of the coolest carbon stars should be reconsidered by taking into account the effect of opacity sources due to polyatomic molecules. In the infrared flux method, however, some uncertainties in model atmospheres may not be very important, since we have applied this to the spectral region least disturbed by line absorption and hence the predicted flux may be relatively free from some shortcomings of the model atmospheres.

Our new effective temperature scale reveals that the C-classification for carbon stars, which has widely been used in the past, can no longer be regarded as a temperature classification scheme, at least for most N-type stars classified as C4-C7. The reason why C-classification has failed to represent the temperature sequence of carbon stars is not clear, but some possibilities were suggested by Bergeat et al. (1976c); the major classification criterion of C-classification is sodium D lines, but as these lines are in the strong CN bands, the D-line index may be more sensitive to CN blanketing rather than to temperature. Also, D-line index may be susceptible to contamination by interstellar or circumstellar D lines. Scalo (1973) considered a possible effect of $(C - O)/H$ ratio on the atmospheric structure and hence on some spectral characteristics including D lines. Certainly, these problems should be examined in more detail. On the other hand, the spectral classification scheme proposed by Richer (1971) shows better correlation with the effective temperatures and his system will provide a promising basis for future spectral classification of cool carbon stars.

In the present paper, carbon stars classified as C0-C3 or R-type stars are not discussed, but these carbon stars are intrinsically different from N-type stars; for example, these relatively hot carbon stars are much less luminous than the cool carbon stars in general and may have quite different evolutionary history. For this reason, it is probably not justified to combine R and N-type stars in a single sequence of C-classification. Also, carbon stars that have been classified as C8-C9 are mostly peculiar stars or Mira-type variables, and the placement of these stars in a classification scheme of carbon stars is an open question. As N-type stars are generally cooler than M-giant stars and also as R-type and N-type stars are different not only in temperature but also in luminosity and evolutionary state, the Harvard classification scheme may be better interpreted as follows:

\[
\begin{align*}
O & \rightarrow B \rightarrow A \rightarrow F \rightarrow G \rightarrow K \rightarrow M \\
& \quad \downarrow R \quad N
\end{align*}
\]

The difficulty in the spectral classification of carbon stars introduced considerable confusion in the interpretation and analysis of the spectra and other observations of
these stars in the past. We believe that the correct temperature sequence and reasonable estimation of the effective temperatures for cool carbon stars can now be given. It is hoped that these findings will provide a new background to future studies of carbon stars both observational and theoretical.

Acknowledgements

The author would like to thank D. E. Blackwell for helpful correspondence and encouragement throughout this work, to S. T. Ridgway for making available his latest preprint on angular diameter measurement of carbon stars and to Y. Yamashita for useful discussion on the spectral classification of carbon stars.

The computation has been carried out at the computer centre of the Tokyo Astronomical Observatory.

References

Intrinsic properties of carbon stars. I