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How High Can A Mountain Be ?

P. A. G. Scheuer *Cavendish Laboratory, Cambridge, CB3 OHE, UK*

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Abstract. The possible height of a mountain on a solid self-gravitating object such as a planet or a neutron star is limited by the strength of the rock. Estimates of the limiting height and conditions for their validity are discussed.

Key words: mountains—planets—neutron stars

1. Formulation of the problem

It is obvious that a mountain of comparable height and base (Fig. 1 a, b), cannot be higher than h_1 times a numerical factor of order unity, where

$$h_1 \simeq 4Y/\rho g. \tag{1}$$

At this point the yield stress Y is reached on a diagonal plane through the mountain. Here ρ is the density of the rock and g is the acceleration due to gravity. For a mountain of square cross-section the numerical factor has a value 4. It is much less obvious what limits the height of a 'hill' (Fig. 1 c), defined as a structure with the

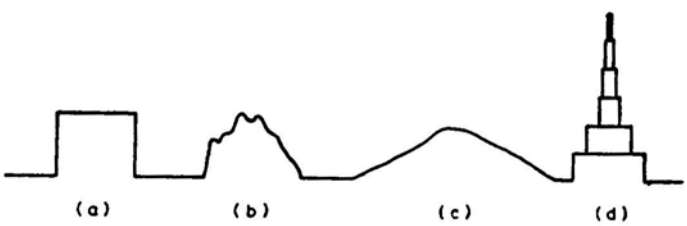

Figure 1. Profiles of four different kinds of mountains: (a) a mountain of square cross-section of comparable height and base (b) mountains of comparable height and base (e) a 'hill' with gently sloping sides (d) composite mountain of arbitrary total height.

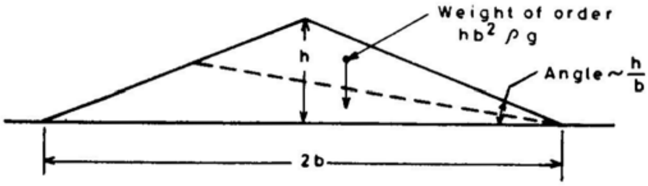

Figure 2. Diagram of a hill to illustrate the derivation of inequality (2).

base $2b$ much greater than the height h. An argument analogous to the above (*cf.* Fig. 2) shows that a solid rock will slide down the hillside if

$$h\, b^2\, \rho g\, (h/b) \gtrsim Y\, b^2$$

i.e.

$$h \gtrsim h_2 = (Y/\rho\, g)^{1/2}\, b^{1/2}. \tag{2}$$

The limiting height h_2 is thus the geometric mean of h_1 and b. Indeed, equation (2) is quoted in the literature (*e.g.* Elder 1976) and applies when the strength of a mountain is limited by a weak stratum in gently sloping sedimentary rock—a case not uncommon on earth. However, the simple argument below shows that it is not a sufficient condition in the simplest case of a mountain made of a homogeneous rock.

Consider a long hill, *i.e.* a ridge, of uniform cross section as shown in Fig. 3 and consider moments about the point P. The net couple due to gravity on the shaded section is (for $h \ll b$, and per unit length of ridge)

$$\int_0^b (x/b)\, h\, dx.\, \rho\, g.\, x = \tfrac{1}{3}\, \rho\, gh\, b^2,$$

and this must not exceed the maximum couple that can be sustained by shear stress as the boundary. $(\pi + h/b)\, b \cdot Y \cdot b \simeq \pi\, Y\, b^2$. Hence

$$h < 3\pi Y/\rho\, g = (3\, \pi/4)\, h_1 \tag{3}$$

where the numerical factor $3\, \pi\, /4$ is an upper bound to the true limit, as only one mode of fracture has been considered.

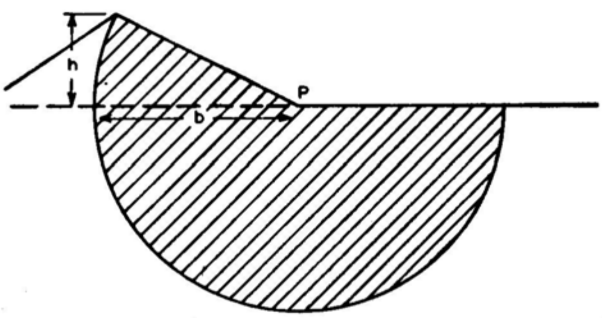

Figure 3. Diagram of a hill to illustrate the derivation of inequality (3),

Whether the same restriction applies to mountains on an inhomogeneous earth is an open question. The argument leading to condition (3) depends on homogeneity to a depth b below the plain level; in essence, it says that the underlying rock cannot support a heavier mountain. The support problem vanishes if the hill either rests on a rigid horizontal base, or (perhaps more realistically) is made of lighter rock floating in roughly isostatic conditions on a denser substrate. Yet, a fracture of the kind shown in Fig. 4(a) can still occur (implying $h \lesssim h_1$) unless the rock is firmly constrained laterally. The horizontal force due to the excess pressure of the heavier surrounding rock on a ‘ floating ‘ mountain is insufficient to prevent this type of fracture for heights much in excess of h_1; this may be illustrated using the model shown in Fig. 4(b). The weight of the triangular section ABC must equal the combined vertical components of the normal reaction P and the shear force S over BC, *i.e.*

$$\tfrac{1}{2}(d+h)^2 \rho g = (P+S)/\sqrt{2}. \tag{4}$$

Also, the magnitude of the shear force is limited by the condition

$$S \lesssim \sqrt{2}\,(d+h)Y. \tag{5}$$

Equating the horizontal forces on BCDE, we have

$$\tfrac{1}{2} d^2 \rho_0 g = (P-S)/\sqrt{2}, \tag{6}$$

While isostatic balance requires

$$d\rho_0 = (d + \alpha h)\rho, \qquad 0 < \alpha < 1 \tag{7}$$

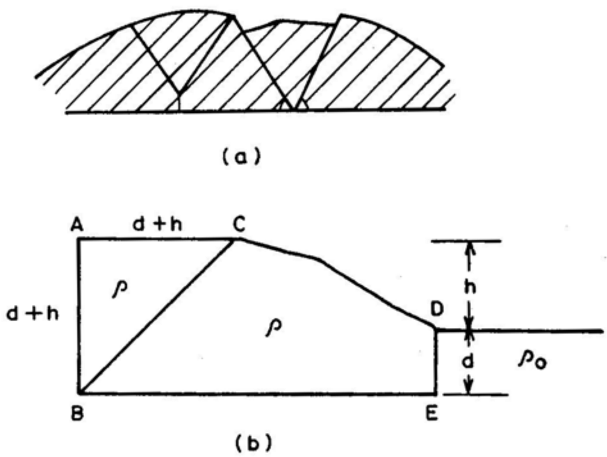

Figure 4. (a) Schematic illustration of a fracture. (b) Model of a mountain of density ρ floating on a substrate of density ρ_0. The height h above the plain level and the depth d below it are also shown. The condition for the stability of the mountain against a fracture along BC is derived in the text.

Here a is a number that depends on the shape of the hill. Subtracting equation (6) from equation (4) we find

$$\tfrac{1}{2}(d+h)^2 \rho g - \tfrac{1}{2} d^2 \rho_0 g = \sqrt{2}\, S$$

Substituting the value of ρ_0 from equation (7) the condition (5) yields

$$\tfrac{1}{2} \rho g (d+h)\left[d+h-\frac{d(d+\alpha h)}{d+h}\right] \leqslant \sqrt{2}\left\{\sqrt{2}(d+h)\, Y\right\}$$

$$h + d\frac{h(1-\alpha)}{d+h} \leqslant 4\, Y/\rho g$$

i.e. $h < h_1$ (8)

since $\alpha < 1$.

On the other hand, if the mountain is firmly glued to a rigid horizontal base, or to the underlying rock below BE in Fig. 4(b), there seems to be no reason why mountains should not form with heights up to $\sim h_2$.

While we have shown that one cannot build mountains much higher than h_1 by placing a homogeneous rock on a broad base, Dr. P. Young (personal communication) has shown that one can, in principle, build mountains of arbitrary height by making them steep enough. Young's mountain has a smooth exponential profile. The essence of the process is to pile mountains, each of height $\sim h_1$, on one another with the bases diminishing in geometric progression so that the total weight above each level is less than Y times the cross-section at that level (Fig. 1d). While such mountains are in equilibrium, the equilibrium becomes unstable if they are too slender to fulfil the Euler condition for the stability of a strut (*e.g.* Marks 1947),

$$P < \tfrac{1}{4} \pi^2 EI/L^2.$$

Here p. is the longitudinal force on a strut of length L, Young's modulus E, and moment of inertia I. If we approximate a uniform vertical bar by a light bar of square cross-section and side a and assume that the weight is equally divided between the top and the bottom, the Euler condition becomes

$$a^2 h_1^+ > (24/\pi^2)\, L^3, \tag{9}$$

where $h_1^+ = E/\rho g$. If $E \simeq Y$ (as in an ideal solid) then $h_1^+ \simeq h_1$ and the condition (9) shows that a stable mountain taller than its base cannot be taller than h_1 .For real solids $E \gg Y$, and the maximum height of a stable steep mountain is then roughly h_1 In $(E/Y) \simeq 5h_1$ While the Euler condition is valid only for long thin struts, the above result should give the correct order of magnitude.

2. Numerical values

Taking the shear strength of rock to be $1{\cdot}5\times 10^6$ kg m^{-2} (typical of values quoted for granite) and density $2{\cdot}65 \times 10^3$ kg m^{-3}, we obtain h_1 = 2250 m for terrestrial mountains, 14000 m for lunar mountains, and 6000 m for Martian mountains. The highest mountains on earth, reach ~ $4h_1$; since isostasy is known to occur in the earth's crust, this is hardly surprising, but we note that the Tibetan plateau, for example, nowhere rises to heights comparable with the theoretical maximum $h_2 \simeq$ 45 km corresponding to its 1000 km horizontal extent. In making these comparisons we make no pretensions to serious geophysics; we merely wish to show that the simple considerations presented here lead to sensible orders of magnitude.

On the surface of a typical neutron star, the density is believed to be ~ 10^8 kg m^{-3} and the surface gravity ~10^{11} m s^{-2}. The strength of the surface material is much harder to estimate. Irvine (1978) gives a Young's modulus of ~10^{18} N m^{-2}, but more recent estimates of the binding energy are an order of magnitude below the values used by Irvine (Baym and Pethick 1979) and his estimate of Young's modulus should thus be reduced to ~10^{17} N m^{-2}. In an ideal solid the shear strength would be similar to Young's modulus, but in real terrestrial solids it is two or three orders of magnitude lower, leading to estimates of 10^{14} to 10^{15} N m^{-2}. Direct scaling from the density, chemical binding energy, and shear strength of granite gives similar results. Thus, on a neutron star, we estimate h_1 = 0·04 to 0·4 mm, with a possible upward revision if neutron star crust behaves more like an ideal solid than does terrestrial rock. We must also bear in mind that the density and other properties of neutron star crust begin to change substantially at depths ~1 m; so far as I am aware the reader is still free to speculate on possible inhomogeneities in composition (and hence possible isostasy) at lesser depths.

3. Conclusions

In a homogeneous rock stable mountains cannot rise much further than $h_1 = Y/\rho g$ above the level of the surrounding plains. Gently sloping 'hills' of crustal rock, 'floating' in more or less isostatic conditions on denser material, may be able to rise to greater heights of the order of $h_2 = (h_1 b)^{1/2}$ where b is the base of the ' hill '.

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